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CHEAP TALK, NEOLOGISMS, AND BARGAINING

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July 1988

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Cheap Talk, Neologisms, and Bargaining*

by

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July 1988

Abstract

This paper shows that in some bargaining games, cheap talk must matter: the unique neologism-proof equilibrium involves informative talk. A simple tradeoff makes both equilibrium talk and neologisms credible: by saying that he is interested in trading, the buyer encourages the seller to continue to bargain but receives poorer terms of trade if trade occurs. All types of both parties are (weakly) better off in the neologism-proof equilibrium than in the equilibrium without talk. Thus, cheap talk before bargaining will happen and is socially desirable.

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1. Introduction

Many game-theoretic models of bargaining under asymmetric information are based on Spence's (1973) signaling model: a bargainer incurs costs (typically through delay) to signal that he is not desperate to trade and so should receive favorable terms of trade if trade occurs.[1] Outside bargaining theory, Crawford and Sobel (1982) have shown that communication is possible without costly signals: costless messages (cheap talk) can be credible if there is some common interest between the parties.

Farrell and Gibbons (forthcoming) and Matthews and Postlewaite (forthcoming) show that cheap talk can matter in a specific bargaining game, a double auction. Farrell and Gibbons show that each bargainer faces a tradeoff that can make his talk credible: by expressing an interest in trading, one bargainer encourages the other to continue to bargain but receives poorer terms of trade if trade occurs; in equilibrium, only buyers (sellers) with high (low) reservation prices are willing to express such an interest in trading. Matthews and Postlewaite show that talk can coordinate the bargainers on different double-auction equilibria (each of which would have been an equilibrium without talk) depending on the bargainers' announced reservation prices, and that such pre-play communication dramatically enlarges the set of equilibrium outcomes of a double auction. Both papers show that talk can matter by constructing equilibria that could not be equilibria without talk.

In this paper we show that in some bargaining games cheap talk not only can but must matter, in the sense that the unique neologism-proof equilibrium (see Farrell (forthcoming)) could not be an equilibrium without talk. In this equilibrium, cheap talk is credible for the same reason we identified in our earlier paper: a bargainer can use talk to encourage his opponent to continue to bargain, but will receive poorer terms of trade if trade occurs. This tradeoff leads to two results in the game we analyze: (1) buyers with high reservation prices prefer to reveal themselves through talk rather than to masquerade

[1]The large literature on this subject begins with Fudenberg and Tirole (1983) and Sobel and Takahashi (1983).

as buyers with low reservation prices, and (2) buyers with high reservation prices prefer to reveal themselves rather than to pool. The first of these results implies that an informative cheap-talk equilibrium exists, and the second establishes that equilibria in which talk is absent or uninformative are not neologism-proof.

We also consider the welfare implications of cheap talk before bargaining. In the game we analyze, all types of both parties are (weakly) better off in the neologism-proof equilibrium than in the equilibrium without talk. Thus, cheap talk before bargaining not only will happen but is socially desirable. This contrasts with our findings for a double auction that some types of both parties do better in an equilibrium with informative talk, but some types do worse, and the ex-ante expected gains from trade are lower with talk than without.

The body of the paper is organized as follows. In Section 2 we develop a variation on the familiar take-it-or-leave-it bargaining model. Our model may be of independent interest as an analysis of trade in idiosyncratic goods, but we have developed it mainly to provide an uncluttered setting for our analysis of cheap talk. In Section 3 we characterize the neologism-proof equilibria in our bargaining game. Section 4 concludes.

2. The Model

We analyze a very simple bargaining game in which a seller, at cost c, can make a single take-it-or-leave-it offer to a buyer. The seller's (privately known) valuation for the good in question is denoted by s, the buyer's by b. If trade occurs at price p then the seller's payoff is p - s - c and the buyer's is b - p. If the seller makes an offer that the buyer rejects then the seller's payoff is -c and the buyer's is zero. Finally, if the seller makes no offer then each player's payoff is zero. Both parties are risk-neutral.[2]

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[2]Perry (1986) shows that the unique perfect Bayesian equilibrium of this game is identical to the unique perfect Bayesian equilibrium of the infinite-horizon alternating-offers game with no discounting in which (i)

The seller's willingness to make an offer and the price she demands if she does so depend on her valuation and on her belief about the buyer's valuation. We assume that it is common knowledge that s and b are independently drawn from a uniform distribution on [0,1]. We ask whether cheap talk by the buyer can influence the seller's belief about b, and so influence both the seller's decision to make an offer and the price she demands if she makes an offer.

Our previous paper shows that, in a double auction, the buyer faces a simple and intuitive tradeoff that can make cheap talk matter in a double auction. To study this tradeoff in our take-it-or-leave-it bargaining game, let u([x,y],b) be the expected payoff to a buyer of valuation b when the seller believes that the buyer's valuation is uniformly distributed on [x,y], an interval contained in [0,1]. (Note that b need not be in [x,y] for this expectation to be well defined.) In order for talk to be credible in equilibrium, there must exist a valuation $b^* \in (0,1)$ such that buyers with valuations above b^* prefer to have the seller believe that their valuation is high while those below b^* prefer her to believe that it is low:

(1) $u([b^*,1],b) \geq u([0,b^*],b)$ for all $b \in (b^*,1]$, and

(2) $u([0,b^*],b) \geq u([b^*,1],b)$ for all $b \in [0,b^*)$.

In the take-it-or-leave-it bargaining game, however, the fact that the seller has all the bargaining power makes it impossible to satisfy both of these inequalities, except in an uninteresting way: $u([0,b^*],b) = 0$ for all $b \in [0,b^*]$.[3] Thus, in this game, the only buyers willing to admit to having low valuations are those who do not trade following talk.

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the seller has the opportunity to make the first offer, (ii) the seller incurs cost c each time she makes an offer, and (iii) the buyer incurs cost $C > c$ each time he makes an offer.

[3]Formally, $u([b^*,1],b^*) = 0$ because given this belief the seller will not make an offer less than b^*. Therefore, $u([b^*,1],b)$ is arbitrarily close to zero for b sufficiently close to (but greater than) b^*. Note that $u([x,y],b)$ is weakly increasing in b, so $u([0,b^*],b^*) = 0$, else (1) fails for b slightly greater than b^*. But this implies that $u([0,b^*],b) = 0$ for all $b \leq b^*$, so (2) cannot hold strictly.

In many other bargaining games, it is not true that the lowest-valuation buyer has an expected payoff of zero.[4] It will become clear below, however, that the simplicity of the take-it-or-leave-it bargaining game enables us to derive crisp results about how talk will affect the bargaining. Rather than abandon the take-it-or-leave-it bargaining game, therefore, we modify its information structure so that the lowest-type buyer has a positive expected payoff. In the process, we develop a model that may be of independent interest as a description of an important class of bargaining problems that has not yet received much attention.

We assume that the buyer does not learn his exact willingness to pay for the seller's good until she defines it precisely. Such imprecision arises when the good in question is complex or idiosyncratic: if the seller must carefully define its specifications (or if the seller must write a contract covering many contingencies that might arise during a long-term relationship with the buyer), the buyer cannot precisely know his valuation beforehand. Our assumption is natural given that the buyer and the seller are bargaining rather than trading in a competitive market at a market-determined price: when the seller's good (or the buyer's need) is idiosyncratic, there is a matching problem as well as a pricing problem, so competition disappears and bargaining results.

We model the buyer's imprecise knowledge of his valuation as follows. First, nature draws the valuations s and b, as above. Second, the seller learns her valuation, s, and the buyer learns what we will call his <u>type</u> (as distinct from his valuation). The buyer's type (t) is either high (H) or low (L), where $t = H$ for $b \in [1/2,1]$ and $t = L$ for $b \in [0,1/2)$. Thus, after learning his type, the buyer's belief about his valuation is uniform on $[0,1/2)$ if $t = L$ and uniform on $[1/2,1]$ if $t = H$.

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[4]In Chatterjee and Samuelson's (1983) linear equilibrium in a double auction, for instance, if s is distributed on [x,y] and b is (independently) distributed on [w,z], then the buyer with valuation w earns a strictly positive expected payoff provided w is sufficiently greater than x. Also, in infinite-horizon games based on Rubinstein's (1982) model with discounting, such as Chatterjee and Samuelson (1987) and Cramton (1984), if there are gains from trade then both parties earn a positive expected payoff.

After learning his type, the buyer can send a costless message (cheap talk) to the seller. The message space is unrestricted, but there is not much that can be said. In fact, it is sufficient to consider the space of equilibrium messages {"H", "L", "pool"}.[5]

The remainder of the game is a take-it-or-leave-it bargaining game. The seller chooses whether to make an offer, and if so what price to demand. If she makes an offer, the buyer learns his valuation, b, and then chooses to accept or reject the offer. If the seller does not make an offer, the game ends, without the buyer learning b. The payoffs are exactly as described above because the messages introduced here are costless.

Two special features of this model are worth noting. First, cheap talk cannot communicate anything other than the buyer's type, t: because the buyer learns b after the seller makes the offer, there is no scope for the buyer to talk about b, and in this take-it-or-leave-it game there is no role for talk by the seller about her valuation, s.[6] Second, the model has moving support: if the buyer learns that t = H, say, then he knows that b is certain to exceed 1/2. Relaxing these assumptions would complicate the analysis but seems unlikely to affect the basic ideas presented below.

3. Analysis

Our analysis proceeds by comparing six expected payoffs, analogous to the expected payoffs u([x,y],b) but based on the buyer's knowledge of his type but not his valuation. These six expected payoffs are denoted U("H", t), U("L", t), and U("pool", t)

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[5]This is in contrast to our earlier paper, in which we considered only the restricted message space {"keen", "not keen"} and so could not completely characterize the equilibrium role of cheap talk in a double auction. Also, note well the distinction between the equilibrium messages {"H", "L", "pool"} and the unrestricted space of possible messages; Farrell describes the large set of potential out-of-equilibrium messages involved in the definition of neologism-proofness.

[6]In this take-it-or-leave-it game, talk by the seller about her valuation could matter only by inducing the buyer to send a subsequent cheap-talk message that varies with the seller's talk. But no matter what her valuation, the seller prefers to have the buyer reveal more rather than less before she has to decide whether to make an offer. Thus, independent of her valuation, the seller would send the message that induces the buyer to reveal as much as possible, so there is no reason for the buyer's talk to vary with the seller's: the seller's talk will be uninformative.

for t ε {H,L}. The first and second of these are the expected payoff to a buyer of type t given that the seller believes that t = H and t = L, respectively. The third is the expected payoff to a buyer of type t given that the seller holds the (prior) belief that each of the two types is equally likely.[7]

We are interested in two (perfect Bayesian) equilibria of our game: a separating equilibrium (in which the buyer's talk perfectly communicates his type to the seller), and a pooling equilibrium (in which the buyer's talk communicates nothing). In each kind of equilibrium, the post-talk bargaining behavior is the unique sequentially rational outcome of the take-it-or-leave-it bargaining game given the seller's belief.[8]

A basic result in the theory of cheap-talk games is that a pooling (or "babbling") equilibrium always exists. We therefore ask whether a separating equilibrium exists, and also whether the equilibria that exist are reasonable---that is, satisfy a refinement. Formally, a separating equilibrium exists if

(3) U("H", H) ≥ U("L", H) and

(4) U("L", L) ≥ U("H", L).

The refinement we use is Farrell's notion of neologism-proof equilibrium. (Many other refinements, such as those developed for signaling games by Cho and Kreps (1987) and Banks and Sobel (1987), have no effect in cheap-talk games.) We say that the pooling equilibrium is not neologism-proof (and so reject it as an equilibrium) if

(5) U("H", H) > U("pool", H) and

(6) U("H", L) ≤ U("pool", L).

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[7]The payoff function u([x,y],b) is of course related to these six expected payoffs: for instance, U("H", L) = E{u([1/2,1],b) | 0 ≤ b < 1/2}.

[8]Note that this uniqueness of sequentially rational post-talk bargaining behavior rules out equilibria in which cheap talk matters for the coordination reason emphasized by Matthews and Postlewaite.

The idea behind these conditions can be phrased in terms of a speech like those popularized by Cho and Kreps. Suppose the pooling equilibrium is to be played and the buyer learns that his type is high, t = H. Then the buyer should deviate from the equilibrium by making the following speech to the seller during the cheap-talk phase: "I claim my type is high. Note that if you believe this claim then I will be (strictly) better off than I would have been in equilibrium, by (5). Note also that if my type were low then I would have no (strict) incentive to get you to believe this claim, by (6)." In Farrell's terminology, (5) and (6) define a credible neologism by the high type; if (5) and (6) hold, we say that a credible neologism by the high buyer-type breaks the pooling equilibrium.

A credible neologism by the low buyer-type also could break the pooling equilibrium, if

(7) U("L", L) > U("pool", L) and

(8) U("L", H) ≤ U("pool", H).

Proposition 1 below implies that if a credible neologism by the low buyer-type breaks the pooling equilibrium then a credible neologism by the high buyer-type also breaks the pooling equilibrium, so we ignore neologisms by the low buyer-type in what follows.

If a separating equilibrium exists, then we also are interested in whether it is neologism-proof. Here the relevant neologism is pooling (the appropriate speech begins "I am not going to reveal my type. Notice that regardless of my type, I have reason to make this deviation..."). The separating equilibrium is not neologism-proof if and only if

(9) U("pool", H) > U("H", H) and

(10) U("pool", L) > U("L", L).

Note that both inequalities are strict, by analogy with (5) and (7). A basic result for two-type cheap-talk games is

Lemma 1. If a separating equilibrium exists and the pooling equilibrium is not neologism-proof then the separating equilibrium is neologism-proof.

Proof. If the pooling equilibrium is not neologism-proof then at least one of (5) and (7) must hold, hence at least one of (9) and (10) must fail. Q.E.D.

We now characterize the neologism-proof equilibria of our model. The analysis rests on a series of simple lemmas (proofs of which are relegated to the Appendix).

Lemma 2. Suppose the seller of valuation s believes that the buyer's valuation is uniformly distributed on [x,y]. Then if the seller makes an offer, the price demanded is $p(s|x,y) = \max\{x,(y+s)/2\}$.

Lemma 3. For all values of c, U("H", L) = 0 and U("pool", L) = 0.

Lemma 4. (a) U("H", H) > 0 if and only if $c < 1/2$.

(b) U("pool", H) > 0 if and only if $c < 1/4$.

(c) U("L", L) > 0 if and only if $c < 1/8$.

(d) U("L", H) > 0 if and only if $c < 1/8$.

Lemma 5. For $c \in (0,1/2)$, U("H", H) > U("pool", H).

Combining these Lemmas (and adding a small argument given in the Appendix) yields the following characterization of the neologism-proof equilibria of our model.

Proposition 1.

(a) If $c > 1/2$ then the unique equilibrium is pooling and it is neologism-proof.

(b) If $c \in (0,1/2)$ then a credible neologism by the high buyer-type breaks the pooling equilibrium. Further, there exists a critical cost level $c^* \in (0,1/8)$ such that when $c \in (c^*,1/2)$ a separating equilibrium exists and is neologism-proof, but when $c \in (0,c^*)$ a separating equilibrium does not exist (and so there does not exist a neologism-proof equilibrium).

Two features of this result are worth noting. First, by Lemma 4(b), if $c \geq 1/4$ then the seller does not make an offer in the pooling equilibrium, no matter what her valuation. Thus, by Proposition 1(a), the pooling equilibrium is neologism-proof only if nothing happens! And second, assuming $c \in (c^*,1/2)$ so that the separating equilibrium exists, both parties are better off in the separating than in the pooling equilibrium: Lemma 5 establishes this for the high buyer-type, while Lemmas 3 and 4(c) do so for the low, and the seller is better off (no matter what her valuation) simply because she has more information.[9] The real importance of Proposition 1, however, is that it leads to our main result:

Proposition 2. For $c \in (c^*,1/2)$, the post-talk bargaining behavior in the unique neologism-proof equilibrium could not be equilibrium behavior without talk.

The proof of this Proposition is simple: the unique equilibrium without talk is pooling; the unique neologism-proof equilibrium for $c \in (c^*,1/2)$ is separating; and the post-talk bargaining behavior in these two equilibria differs in many obvious ways. We now turn to

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[9]It is of course not generally true in games that having more information makes a player better off. In our bargaining game, however, the buyer's optimal action depends only on the price the seller demands and on b, and so is independent of what the buyer thinks the seller knows. In essence, this reduces the game to a single-person decision problem for the seller, in which case it is true that more information is better.

a detailed discussion of these differences in post-talk bargaining behavior in order to interpret the effects of cheap talk in our model.

There are two ways in which post-talk bargaining behavior can differ among equilibria: in the seller's decision to make an offer, or in the price she demands (or both). Our main results involve both of these differences: the high buyer-type is willing to pay a higher price if trade occurs in order to increase the probability that trade will occur, and the low buyer-type strictly prefers not to make this tradeoff.

In some of the equilibria considered in Proposition 2, however, simpler (and, we think, less interesting) arguments suffice to support the equilibrium. In some cases, for instance, the low buyer-type is indifferent about his message: (4) holds with equality. Also, in some cases the price the seller demands if she makes an offer is the same in the separating and pooling equilibria, so the equilibria differ only in the probability that the seller makes an offer. We now identify the equilibria described in Proposition 2 that are relatively uninteresting for either of these reasons; this leaves us with the equilibria that precisely embody the intuition we have described.

Consider first the pooling equilibrium. Lemma 2 implies that if the seller with valuation s makes an offer then the price she demands is $p(s|0,1) = (1+s)/2$. Straightforward calculation then shows that (assuming $c < 1/4$) the sellers with valuations $s < 1 - 2\sqrt{c}$ make offers; as noted above, when $c \geq 1/4$ no seller-type makes an offer.

Now consider the separating equilibrium. When $c \in (1/8,1/2)$, more seller-types make offers in the separating equilibrium than in the pooling equilibrium, but those types that make offers in both equilibria make the same offer in each equilibrium.[10] The fact that more seller-types make offers in the separating equilibrium is most apparent when

[10]It is only by accident that the price that a seller with valuation s demands in the pooling equilibrium, $p(s|0,1) = (1+s)/2$, is the same as the price that she demands in the separating equilibrium when the buyer claims to be the high type, $p(s|1/2,1) = (1+s)/2$. If the high buyer-type consisted of buyers with valuations $b \in [b',1]$ for some $b' > 1/2$, then the price a seller with valuation s demands in the separating equilibrium when the buyer claims to be the high type would be $p(s|b',1) = \max(b',(1+s)/2)$, which differs from the price she demands in the pooling equilibrium.

$c \in (1/4,1/2)$: the seller makes no offer in the pooling equilibrium, but in the separating equilibrium, with probability 1/2 the buyer announces that his type is high, in which case the seller updates her belief about his valuation b, and, if s is low, makes an offer. An analogous argument applies when $c \in (1/8,1/4)$, except that in this case very-low-valuation sellers make offers in both equilibria.

Consequently, when $c \in (1/8,1/2)$, saying "H" strictly dominates saying "pool" or "L" for the high buyer-type and weakly dominates these alternatives for the low buyer-type. For these values of c, therefore, the existence of a separating equilibrium depends critically on the weak inequality in (4). A more interesting equilibrium exists when talk not only determines which seller-types make offers but also what prices they demand. This occurs when $c \in (c^*,1/8)$, as the following table makes clear.

	Price Demanded by Seller of Valuation s	Highest Valuation Seller to Make an Offer
Pooling Equilibrium	$p(s\|0,1) = \frac{1+s}{2}$	$s^*(c\|\text{"pool"}) = 1-2\sqrt{c}$
Separating Equilibrium following "H"	$p(s\|1/2,1) = \frac{1+s}{2}$	$s^*(c\|\text{"H"}) = 1-\sqrt{2c}$
Separating Equilibrium following "L"	$p(s\|0,1/2) = \frac{1+2s}{4}$	$s^*(c\|\text{"L"}) = \frac{1-2\sqrt{2c}}{2}$

Thus, a low-valuation seller, for instance, makes offers in both the pooling and separating equilibria but demands a different price in the latter because the buyer's talk has influenced her belief about the buyer's valuation, b.

4. Conclusion

The first result of our analysis is that the pooling equilibrium (the only equilibrium in which talk does not matter) is neologism-proof only if the cost of making an offer is so high that no offer is made by a seller of any valuation. We can rephrase this more strikingly: restricting attention to neologism-proof equilibria, if an offer (not to mention trade) is observed, then talk must have played a role. The force behind this result is the natural and credible message available to the high-type buyer: when $t = H$, the buyer encourages the seller to make an offer, at some cost in the terms of trade if trade occurs, and this breaks the pooling equilibrium. We expect analogous cheap-talk messages to be credible in many other bargaining games, so that pooling equilibria in these games often will not be neologism-proof; we will pursue this idea in future work.

Our second result is that when $c \in (c^*, 1/2)$, the separating equilibrium not only exists but is the unique neologism-proof equilibrium. In this equilibrium, talk affects whether and at what terms trade takes place. Combined with our first result, this shows that in some bargaining games (i.e., for some values of c) talk not only can matter but must matter, in the sense that the only neologism-proof equilibrium could not be an equilibrium without talk. We find this result particularly appealing when $c \in (c^*, 1/8)$, in which case both buyer -types have a strict incentive to reveal themselves in a separating equilibrium and both buyer-types have a strict incentive to reveal themselves rather than to pool.

Our final result is that, in the game we analyze, all types of both parties are (weakly) better off in the neologism-proof equilibrium than in the equilibrium without talk. Thus, cheap talk before bargaining not only will happen but is socially desirable.

APPENDIX

Proof of Lemma 2. The seller's problem is to choose p to solve

$$\max_{p} \quad s\,F(p) + p\,[1 - F(p)],$$

where $F(p) = (p - x)/(y - x)$ for $p \in [x,y]$, is zero for $p < x$, and is one for $p > y$. The first-order condition yields $p = (y+s)/2$, which is a global maximizer provided that $F(p)$ is strictly positive at this price. Q.E.D.

Proof of Lemma 3. By Lemma 2, the minimum price demanded comes from the zero-valuation seller and so is

$$p(0|x,y) = \max\{x, y/2\}.$$

Thus, in U("H", L), where the seller believes that b is uniformly distributed on [1/2,1], the minimum price is 1/2, and in U("pool", L), where the seller believes that b is uniformly distributed on [0,1], the minimum price also is 1/2. In neither case does the low-type buyer receive an acceptable offer. Q.E.D.

Proof of Lemma 4. Each case is solved by computing the value of c such that the zero-valuation seller is indifferent about making an offer.

(a) In U("H", H), the zero-valuation seller believes that b is uniformly distributed on [1/2,1] and so demands $p(0|1/2,1) = 1/2$ if she makes an offer. This offer is accepted with probability one and so earns a profit for the zero-valuation seller of 1/2. Thus, if $c = 1/2$, the zero-valuation seller is indifferent about making an offer.

(b) In U("pool", H), the zero-valuation seller believes that b is uniformly distributed on [0,1] and so demands $p(0|0,1) = 1/2$ if she makes an offer. This offer is accepted with probability 1/2 and so earns a profit for the zero-valuation seller of 1/4. Thus, if $c = 1/4$, the zero-valuation seller is indifferent about making an offer.

(c) In U("L", L), the zero-valuation seller believes that b is uniformly distributed on [0,1/2] and so demands p(0|0,1/2) = 1/4 if she makes an offer. This offer is accepted with probability 1/2 and so earns a profit for the zero-valuation seller of 1/8. Thus, if c = 1/8, the zero-valuation seller is indifferent about making an offer.

(d) In U("L", H) the zero-valuation seller believes that b is uniformly distributed on [0,1/2], as in (c). This belief leads to the same seller behavior as in (c). The demand p(0|0,1/2) = 1/4 is accepted with probability one by the high-type buyer, but the zero-valuation seller does not expect this behavior from the buyer and so is indifferent about making an offer when c = 1/8. Q.E.D.

Proof of Lemma 5. The result follows from Lemmas 4(a) and 4(b) for $c \in (1/4,1/2)$. The argument for $c \in (0,1/4)$ is as follows. By Lemma 2, the price demanded by the seller of valuation s is identical in U("H", H) and in U("pool", H):

$$p(s|1/2,1) = p(s|0,1) = \frac{1+s}{2}.$$

The only difference between U("H", H) and U("pool", H) is that, for any given value of c, strictly more sellers choose to make an offer in the former, so the high-type buyer is better off. Formally, simple computations show that in U("H", H) the highest-valuation seller to make an offer is $s^*(c|\text{"H"}) = 1-\sqrt{2c}$, while in U("pool", H) it is $s^*(c|\text{"pool"}) = 1-2\sqrt{c}$. The former is strictly larger. Q.E.D.

Proof of Proposition 1.

(a) The separating equilibrium does not exist because U("H", H) = 0, by Lemma 4(a), so (3) fails. Similarly, the pooling equilibrium is neologism-proof because by Lemmas 4(a) and 4(c), (5) and (7) fail.

(b) By Lemma 5, (5) holds, and by Lemma 3, (6) holds. Thus, a credible neologism by the high buyer-type breaks the pooling equilibrium. Therefore, by Lemma 1, if the separating equilibrium exists then it is neologism-proof.

Suppose first that $c \in (1/8,1/2)$. By Lemmas 4(a) and 4(d), (3) holds. By Lemmas 3 and 4(c), (4) holds weakly. Hence a separating equilibrium exists.

Now suppose that $c \in (0,1/8)$. By Lemmas 3 and 4(c), (4) holds strictly. It thus remains to show that (3) holds if and only if $c \in (c^*,1/8)$, for some $c^* \in (0,1/8)$. Using the values of $p(s|0,1/2)$, $s^*(c|\text{"L"})$, $p(s|1/2,1)$, and $s^*(c|\text{"H"})$ given in the text, calculations show that

$$U(\text{"L"}, H) = \frac{(1-4x)(3+4x)}{16} \quad \text{and}$$

$$U(\text{"H"}, H) = \frac{x}{2} + \frac{2(1-2x)^3}{24},$$

where $x = \sqrt{c/2}$. Note that if $c \in (0,1/8)$ then $x \in (0,1/4)$. Thus, $U(\text{"H"}, H) > U(\text{"L"}, H)$ if and only if

$$f(x) = 32x^3 - 96x^2 - 24x + 5 < 0.$$

Note that $f(1/4) < 0$ and $f(0) > 0$. (Both of these are obvious from first principles: consider $c = 1/8$ in Lemmas 4(a) and 4(d) and $c = 0$, respectively.) Finally, note that $f(x)$ is decreasing for $x \in (0,1/4)$, so there exists a unique $x^* \in (0,1/4)$ that solves $f(x) = 0$. Therefore, c^* solves $x^* = \sqrt{c/2}$. Q.E.D.

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